Assignment 1

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Course: Algebraic Topology II (KSM4E02) Instructor: Aritra Bhowmick

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Q1. (*Proper Triad and Summation of Maps*) Let $(X; X_1, X_2)$ be a proper triad with $X = X_1 \cup X_2$, and set $A := X_1 \cap X_2$. Let $f, f_1, f_2 : (X, A) \to (Y, B)$ be maps such that

$$f_1|_{X_1} = f|_{X_1}, \quad f_1(X_2) \subset B \qquad\qquad f_2|_{X_2} = f|_{X_2}, \quad f_2(X_1) \subset B.$$

Show that $f_\star = f_{1\star} + f_{2\star}$.

Hint : Consider the space-level diagram

$$\begin{tikzcd}
(X_1, A) \arrow[dr, "\iota_1"] \arrow[dd, "k_2"'] & & (X_2, A) \arrow[dl, "\iota_2"'] \arrow[dd, "k_1"] \\
& (X, A) \arrow[dl, "j_2"'] \arrow[dr, "j_1"] \arrow[dd, "f"] \arrow[dd, bend right, "f_2"'] \arrow[dd, bend left, "f_1"] & \\
(X, X_2) \arrow[dr, "f_1'"'] & & (X, X_1) \arrow[dl, "f_2'"] \\
& (Y, B) &
\end{tikzcd}$$

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Q2. Let $(X; X_1, X_2)$ be a proper triad. Verify that $(X_1 \cup X_2; X_1, X_2)$ is again a proper triad. Show that the diagram

$$\begin{tikzcd}
H_{n+1}(X, X_1 \cup X_2) \arrow[r, "\partial"] \arrow[d, "\partial_1"'] & H_n(X_1, X_1 \cap X_2) \arrow[d, "\partial_2"] \\
H_n(X_1 \cup X_2) \arrow[r, "\Delta"'] & H_{n-1}(X_1 \cap X_2)
\end{tikzcd}$$

commutes, where ∂ is the boundary map of the long exact sequence of the triad $(X; X_1, X_2)$, and Δ is the boundary map of the Mayer-Vietoris sequence of $(X_1 \cup X_2; X_1, X_2)$. The boundary maps ∂_i are from the long exact sequence of the corresponding pairs. $3 + 7$ = 10

Q3. Let $(X; X_1, X_2)$ be a proper triad. Consider the diagram

$$\begin{tikzcd}
H_{n+1}(X, X_1 \cup X_2) \arrow[r, "\partial_1"] \arrow[d, "\partial_2"'] & H_n(X_1, X_1 \cap X_2) \arrow[d, "\partial"] \\
H_n(X_2, X_1 \cap X_2) \arrow[r, "\partial"'] & H_{n-2}(X_1 \cap X_2)
\end{tikzcd}$$

where ∂_i are the boundary maps of the corresponding triad, and the two ∂'s are the boundary maps of the corresponding pairs. Show that the diagram *anticommutes*, i.e, show that $\partial \circ \partial_1 + \partial \circ \partial_2 = 0$. 10

Q4. Prove the relative Mayer-Vietoris sequence. 10